Alternative Accelerated CC Math 6/7 – UNIT 5
Statistics and Probability

Critical Area: Description of the critical area: Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability.

Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students build on their work with single data distributions to compare two data distributions and address questions about differences between population. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

CLUSTERS	COMMON CORE STATE STANDARDS
Develop understanding of statistical variability.	6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* 6.SP.2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. 6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Summarize and describe distributions.	6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. 6.SP.5. Summarize numerical data sets in relation to their context, such as by: • Reporting the number of observations. • Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. • Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

	• Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
Use random sampling to draw inferences about a population.	7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. 7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
Draw informal comparative inferences about two populations.	7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. 7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
Investigate chance processes and develop, use, and evaluate probability models.	7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. 7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. 7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the

	discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? 7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
STANDARDS IN CCSS MATH 6	PREREQUISITE COMMON CORE STATE STANDARDS
6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*	5.MD.2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
6.SP.2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and	5.MD.2

overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	6.SP.1, 6.SP.2
6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	5.MD.2

6.SP.5. Summarize numerical data sets in relation to their context, such as by:
• Reporting the number of observations.
• Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
  • Giving quantitative measures of center (median and/or means) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with references to the context in which the data were gathered.
  • Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
	6.SP.2, 6.SP.3

7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates any unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and probability near 1 indicates a likely event.	
7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population	6.SP.1, 6.SP.2, 7.SP.5

from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a mutilple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.4.Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh- grade science book are generally longer than the words in a*

7.SP.1

5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product (a/b) x q as a parts of a partition of q into b equal equal parts; equivalently, as the result of a sequence of operations a x q + b. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (a/b) x (c/d) = ac/bd.)*

b. Find the area of a rectangle with fractional side lengths by tilting it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and fraction Products as rectangular areas.

6.SP.2, 6.NS.1

7.SP.2, 7.SP.3

chapter of a fourth-grade science book.

7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled rougly 200 times, but probably not exactly 200 times.*

7.SP.5, 7.RP.3

7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7.SP.6, 7.RP.3

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

7.SP.7, 7.RP.3

a. Understand that, just as with the simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	
MATHEMATICAL PRACTICES	LEARNING PROGRESSIONS
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the arguments of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	**Click on the links below to access common core team's grade 6 learning progressions for this unit on:** **Statistics and Probability** http://commoncoretools.files.wordpress.com/2011/12/ccss_progression_sp_68_2011_12_26_bis.pdf

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
In Grade 6 Students build on the knowledge and experiences in data analysis developed in earlier grades. They develop a deeper understanding of variability and more precise descriptions of data distributions, using numerical measures of center and spread, and terms such as cluster, peak, gap, symmetry, skew, and outlier. They begin to use histograms and box plots to represent and analyze data distributions. As in earlier grades, students view statistical reasoning as a four-step investigative process: • Formulate questions that can be answered with data	How are mean, median, mode and range related? What are the best ways to predict the outcomes of an experiment? Can data be manipulated to show what you want? What kind of conclusions can be made from	Variability, Variation Deviation Set of Data Distribution Data and Plots Histogram Box Plots Mean, median, mode and range Interquartile Range Cluster, peak, gap, symmetry, skew, and outlier Sample space

• Design and use a plan to collect relevant data • Analyze the data with appropriate methods • Interpret results and draw valid conclusions from the data that relate to the questions posed. Such investigations involve making sense of practical problems by turning them into statistical investigations; moving from context to abstraction and back to context; repeating the process of statistical reasoning in a variety of contexts. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.	a set of data, taking into account the numerical measures of center and spread, and terms such as cluster, peak, gap, symmetry, skew, and outlier? How do you determine which measures of variability should be used to draw informal comparative inferences? How are lists, tables, tree diagrams or simulation used to find the probability of an event? How is probability used to predict frequency of an event?	Tree diagrams Prediction Statistsics Probability Simple events Variation Compound events Gauge Simulation Variability Inferences Distribution Deviations Frequency Comparative Random Relative Discrepancy Draw Inference	

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Teaching and Learning Framework http://illuminations.nctm.org/ **NLVM** http://nlvm.usu.edu/ **NCTM Illuminations activities**	• Real world connections • Structured instructional conversations • Use visuals and physical model of shapes • Use of Language frames for classroom communication • Encourage Student to student questioning	SBAC - http://www.smarterbalanced.org/ PARCC - http://parcconline.org/samples/mathematics/grade-6-slider-ruler

7.NS Comparing Freezing Points http://www.illustrativemathematics.org/illustrations/314 7.NS Distances on the Number Line 2 http://www.illustrativemathematics.org/illustrations/310 7.NS Operations on the number line http://www.illustrativemathematics.org/illustrations/46 **California Draft Mathematics Framework:** http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp. 7.NS Operations on the number line http://www.illustrativemathematics.org/illustrations/46 **California Draft Mathematics Framework:** http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp. **LAUSD Adopted Textbooks:** **California Mathematics: Concepts Skills and Problem Solving, Glencoe McGraw-Hill** 7.1 Circumference and Area of Circles 7.2 Problem Solving Investigation 7.3 Area of Complex Figures 7.4 Three-Dimensional Figures 7.5 Volume of Prisms and Cylinders 7.6 Volume of Pyramids and Cones 7.7 Surface Area of Prisms and Cylinders 7.8 Surface Area of Pyramids	• Journal writing prompts (link) • Technology to show visual representations of geometric figures: Geometry sketchpad • Use visuals to illustrate multiple representations of rate of change • Real-world connections • Structured instructional conversations (Think-Pair-Share)	SBAC - http://www.smarterbalanced.org/ PARCC - http://parcconline.org/samples/mathematics/grade-6-slider-ruler

7.9 Similar Solids 11.1 Problem Solving Investigation 11.4 Measures of Central Tendency and Range 11.5 Measures of Variation 11.8 Select an Appropriate Display **California Math: Course 2 - McDougal Littell** 7.NS Operations on the number line http://www.illustrativemathematics.org/illustrations/46 **California Draft Mathematics Framework:** http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp. **California Math: Course 2 - McDougal Littell** 8.5 Triangles and Their Areas 8.7 Quadrilaterals and Their Areas 8.8 Circumferences and Areas of Circles 9.2Translations in the Coordinate Plane 9.3 Reflections in the Coordinate Plane 9.4 Dilations in the Coordinate Plane 10.1 Lines and planes 10.2 Three-Dimensional Figures 10.3 Surface Areas of Prisms and Cylinders 10.4 Surface Areas of Pyramids and Cones 10.5 Volumes of Prisms and Cylinders 10.6 Volumes of Pyramids and Cones 10.7 Similar Solids 11.1 Mean, Median, Mode, and Range		

LANGUAGE GOALS

Students will recognize and write statistical questions. How old am I?" is not a statistical question, but "How old are the students in my school?"
Students will explain the meaning of statistical distribution.
Students will compare and contrast the differences between measure of center for a numerical data and variation.

Example: Measure of central tendency is_____ and variability describes how_____.

- Students will understand the context and relationship between data in order to make prediction and draw inferences.
  Example: Given two different sets of data, I can predict that _____ . Based on this prediction, I could draw inference that _____.

- Students will be able to select the appropriate formulas needed to solve real-world and mathematical problems.
  Example: I can compare the formulas for computing area, surface area, and volume of figures and objects, by _____.

- Students will be able to justify steps taken to arrive at a logical conclusion.
  Example: If the situation is_____, then I can conclude that _____

PERFORMANCE TASKS

Inside Mathematics

Statistics and Probability (SP)

6.SP.1, SP.4, SP.5- Through the Grapevine: Problem of the Month http://insidemathematics.org/problems-of-the-month/pom-throughthegrapevine.pdf

6.SP.2 - Pick a Pocket: Problem of the Month http://insidemathematics.org/problems-of-the-month/pom-pickapocket.pdf

6.SP.3, SP.5 - Baseball Players: Task http://insidemathematics.org/common-core-math-tasks/6th-grade/6-2003%20Baseball%20Players.pdf

Research Development and Accountability

6G – Triangle Try Outs http://www.rda.aps.edu/mathtaskbank/pdfs/tasks/6-8/t68TriTryOut.pdf

Inside Mathematics

6.G.1 Polly Gone: Problem of the Month http://insidemathematics.org/problems-of-the-month/pom-pollygone.pdf

6.G.2 - Building Blocks: Task http://insidemathematics.org/common-core-math-tasks/6th-grade/6-2007%20Building%20Blocks.pdf

MATHEMATICS ASSESSMENT PROJECT

- Maximizing Area: Gold Rush http://map.mathshell.org/materials/lessons.php?taskid=415#task415
- Using Dimensions: Designing a Sports Bag http://map.mathshell.org/materials/lessons.php?taskid=416#task416
- Drawing to Scale: Designing a Garden http://map.mathshell.org/materials/lessons.php?taskid=494#task494
- Applying Angle Theorems http://map.mathshell.org/materials/lessons.php?taskid=214#task214
- Estimating: Counting Trees http://map.mathshell.org/materials/lessons.php?taskid=422#task422
- Estimations and Approximations: The Money Munchers http://map.mathshell.org/materials/lessons.php?taskid=220#task220
- Drawing to Scale: Designing a Garden http://map.mathshell.org/materials/lessons.php?taskid=494#task494

- Evaluating Statements About Probability http://map.mathshell.org/materials/lessons.php?taskid=225#task225
- Evaluating Statements About Probability http://map.mathshell.org/materials/download.php?fileid=701

ILLUSTRATIVE MATHEMATICS

- 7.SP Election Poll, Variation 1 http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/235/original/illustrative_mathematics_235.pdf?1343857065
- 7.SP Election Poll, Variation 2 http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/559/original/illustrative_mathematics_559.pdf?1343857067
- 7.SP Election Poll, Variation 3http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/558/original/illustrative_mathematics_558.pdf?1343857068
- 7.SP Estimating the Mean State Area http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/260/original/illustrative_mathematics_260.pdf?1343857069

DIFFERENTIATION

FRONT LOADING	ACCELERATION	INTERVENTION
• Students apply their understanding of data and how to construct line plots. • Students apply and extend their knowledge of symmetric shapes • Students use their knowledge in division, fractions, and decimals in computing a new measure of center-arithmetic mean, often simply called the mean.	• Relate the area of study to other subjects within, between, and across disciplines. Such as in, • Geography and Environmental Literacy • Students apply knowledge and understanding of data collection to answer questions regarding – the emergence and expansion and decline of civilizations, societies and regions	• Small teacher to student ratio discussion • Emphasize think-pair-share • Make connections to real life • Census data, experimental results • Small teacher to student ratio discussion – have students draw informal comparative inferences about two populations (boys vs. girls) • Emphasize think-pair-share • Data discussed comes from sampling life data (soccer team height vs. football team height) an NOT a textbook • Use Physical objects to demonstrate the math. In geometry : Such as cones, squares, etc • In probability and statistics: Census data, experimental results
• Students Formulate questions that can be answered with data • Students design and use a plan to collect relevant data • Students analyze the data with appropriate methods • Students are able to interpret results and draw valid conclusions from the data that relate to the questions posed. Such investigations involve making sense of practical problems by turning them into statistical investigations; moving from context to abstraction and back to context; repeating the process of statistical reasoning in a variety of contexts.	• LEARN NC Interdisciplinary Math and Social Studies, Gridding an archaeological dig site http://www.learnnc.org/lp/pages/1005 • Provide students with opportunities to be recognized for their previous knowledge and to be allowed to avoid redundant learning by being encouraged to learn the sophisticated and advanced information and skills of the curriculum at their own rate. This also includes the opportunity for students to make personal meeting of the lesson. For example: • Simulating probability experiments via technology where students collect meaningful data (type of	

	music, who eats cafeteria food)	

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